Trails in Quantum Mechanics and Surroundings 2015

Programme

Wednesday 08/07/2015

14:00 - 14:30 Opening

14:30 - 15:15 **Italo Guarneri** – Classical prefiguration of quantum localization

15:15 - 16:00 **Alessandro Zampini** – Dirac-Kaehler operator on spheres

16:00 - 16:30 Coffee Break

16:30 - 17:15 **Domenico Finco** – Schrödinger equation with nonlinear defect as effective model

17:15 - 18:00 **Serena Cenatiempo** – A norm approximation for the many body boson dynamics

18:00 - 18:15 Break

18:15 - 19:00 **Silvestro Fassari** – The Hamiltonian of the harmonic oscillator with an attractive δ'–interaction centred at the origin as approximated by the one with a triple of attractive δ–interactions

Thursday 09/07/2015

09:30 - 10:15 **Sergio Albeverio** – Quantum fields, point interactions, and Casimir effect

10:15 - 11:00 **Lorenzo Tentarelli** – NLS equation on metric graphs with localized nonlinearities

11:00 - 11:30 Coffee Break

11:30 - 11:50 **Lorenzo Pinna** – Controllability of spin-boson systems

11:50 - 12:10 **Emanuela Giacomelli** – On the existence of magnetic time reversal symmetry

12:10 - 12:30 **Giulia Basti** – A remark on the Hamiltonian for three identical bosons with zero-range interactions

12:30 - 14:30 Lunch Break

14:30 - 15:15 **Hynek Kovařík** – Time decay of the wave functions for magnetic Schrödinger operators

15:15 - 16:00 **Ian Jauslin** – Non-perturbative renormalization group in a hierarchical Kondo model

16:00 - 16:30 Coffee Break

16:30 - 16:50 **Daniele Dimonte** – Giant vortex state in fast rotating Bose-Einstein Condensates

16:50 - 17:10 **Giovanna Marcelli** – The analogies between prototypes of periodic Schrödinger operators via Bloch-Floquet methods and the ergodic Laplacian, an ergodic Schrödinger operator

17:10 - 17:30 **Giovanni Antinucci** – Boundary effect at critical point

20:30 Social Dinner

Friday 10/07/2015

09:30 - 10:15 **Enrico Serra** – NLS ground states on graphs

10:15 - 11:00 **Andrea Mantile** – Selfadjoint elliptic operators with boundary conditions on not closed hypersurfaces

11:00 - 11:30 Coffee Break

11:30 - 12:15 **Emanuele Haus** – Growth of Sobolev norms for the non-linear Schrödinger equation on the two-dimensional torus

12:15 - 13:00 **Domenico Monaco** – Stability of closed gaps for the alternating Kronig-Penney Hamiltonian

Titles and Abstracts

Quantum fields, point interactions, and Casimir effect
Sergio Albeverio

Boundary effect at critical point
Giovanni Antinucci

The study of critical phenomena is an important topic both in quantum mechanics and in statistical physics, in particular, understanding the boundary effects at the critical point has raised many interesting questions (E.G. anomalous localization in Luttinger liquids, boundary effects on anomalous exponents in quantum systems, or boundary effects and conformal covariance in statistical critical $2d$ models). For 2-dimensional systems ($2d$ for statistical systems, $1+1d$ for quantum systems) these theories are typically described, at the critical point, by conformal field theories. It would therefore be interesting to derive conformal invariance properties starting from a microscopic description of the system. Our starting point to explore this question is to study the bulk theory of a system of interacting spinless fermions in one dimension, defined on the semispace $\mathbb{Z}^+$ with Dirichlet boundary conditions (DBC) at the origin, using Renormalization Group methods. The main problem is that all the Renormalization group method (developed in the last 40 years by Gallavotti, Benfatto, Mastropietro, Giuliani et al.) depends in a crucial way on the translation invariance of the model. So the first step is to generalize the Renormalization Group methods in a way hopefully robust with respect to boundary conditions.

A remark on the Hamiltonian for three identical bosons with zero-range interactions
Giulia Basti

We consider a model consisting of three identical bosons with two-body zero-range interactions and, starting from the formal Hamiltonian, we construct the corresponding quadratic form. It is well known that such a quadratic form is unbounded from below. On the other hand, we show that the restriction of the form to the subspace corresponding to non zero angular momentum is closed and bounded from below. This implies the stability of the system on that subspace.

A norm approximation for the many body boson dynamics
Serena Cenatiempo

We consider a system of N bosons in three dimensions interacting via a pair potential $N^{3\beta-1}V(N^\beta(x_i-x_j))$, where $0<\beta<1$ and $\mathbf{x}=(x_1,\dots,x_N)$ denotes the positions of the particles. The dynamics of the system is known to be approximated by a cubic nonlinear Schrödinger equation. So far the available results establish the convergence in trace norm of the reduced k-particle density matrices associated with the solution of the many body Schrödinger equation towards products of solutions of the non linear Schrödinger equation.

In this talk we go one step further: on the bosonic Fock space we construct a limiting unitary evolution with a quadratic generator, providing a norm approximation for the full many body dynamics for a certain class of initial data, for any $0<\beta<1$. Obstructions to the extension of this result to the Gross-Pitaevskii scaling limit $\beta=1$ will be also discussed.

Joint work with Chiara Boccato and Benjamin Schlein.

Giant vortex state in fast rotating Bose-Einstein Condensates
Daniele Dimonte

I will talk about phase transitions for a fast rotating Bose-Einstein Condensate. When rotating the Condensate undergoes three phase transitions. In particular the third one occurs when the rotating speed is of order $1/\varepsilon^4$ with respect to ε, the coupling constant; under that speed the Condensate is essentially supported in a ring with vortices in it and over it becomes higher, the vortices disappear and the Condensate distributes itself as in a giant "unique vortex state". I will discuss about an upper bound for this third critical speed using minimizing techniques on some effective functionals.

On the existence of magnetic time reversal symmetry
Emanuela Giacomelli

The time reversal operator for spin 0 and spin $1/2$ particles does not commute with the Hamiltonian that describes their dynamics in the presence of a magnetic field. A possible strategy to circumvent this problem consists in considering a deformation of such operator, usually called "magnetic time reversal operator" (MTR operator). The existence of a magnetic time reversal operator is relevant in the context of gapped band insulators, since it would imply the existence of exponentially localized magnetic Wannier functions. We actually prove a "no-go theorem", under the additional hypothesis that the unitary part of the MTR operator is a multiplication operator.

The Hamiltonian of the harmonic oscillator with an attractive δ'–interaction centred at the origin as approximated by the one with a triple of attractive δ–interactions
Silvestro Fassari

In this note we provide an alternative way of defining the self-adjoint Hamiltonian of the one-dimensional harmonic oscillator perturbed by an attractive δ'–interaction centred at the origin (the bottom of the confining parabolic potential), that was rigorously defined in a previous paper (essentially presented in one of the sessions of TQMS 2013 held in Frascati) by means of a "coupling constant renormalisation". Here we rigorously get the Hamiltonian as a norm-resolvent limit of the harmonic oscillator Hamiltonian perturbed by a triple of attractive δ–interactions.

Joint work with S. Albeverio and F. Rinaldi

Schrödinger equation with nonlinear defect as effective model
Domenico Finco

We discuss the convergence of the evolution for a NLS with rescaled inhomogeneous nonlinearity to a NLS with concentrated nonlinearity in dimension one and dimensione three.

Classical prefiguration of quantum localization
Italo Guarneri

Classical dynamical systems designed so that momentum can only change by multiples of a constant exhibit dynamical behaviors, that mimic the scenario of quantum dynamical localization; namely, strong inhibition of diffusion, or quadratic energy growth, depending on the arithmetic nature of a constant. Classical systems of this type may be produced by inserting piecewise linear, continuous potentials in the Standard Map (e.g., the "sawtooth map") but they also appear in some polygonal billiards. They are believed to be "weakly ergodic", and challenge rigorous mathematical analysis. Such purely classical behavior is in this talk justified by a phase-space generalization of a well known quantum argument by Fishman, Grempel, and Prange, whereby the kicked rotor localization was assimilated to the Anderson localization.

Growth of Sobolev norms for the non-linear Schrödinger equation on the two-dimensional torus
Emanuele Haus

We study the non-linear Schrödinger equation (with analytic nonlinearity of any order) on the two-dimensional torus and exhibit orbits whose Sobolev norms grow with time. The main point is to make use of an accurate combinatorial analysis in order to reduce to a sufficiently simple toy model, which generalizes the one discussed in the paper by J. Colliander, M. Keel, G. Staffilani, H. Takaoka and T. Tao for the case of the cubic NLS. We also give estimates of the time needed to obtain such growth, by refining and adapting to this more general case the techniques used for the cubic case in the work by M. Guardia and V. Kaloshin. This is a joint work with M. Guardia and M. Procesi.

Non-perturbative renormalization group in a hierarchical Kondo model
Ian Jauslin

We consider a hierarchical version of the Kondo model, which consists in a 1-dimensional chain of spin-$1/2$ Fermions interacting magnetically with a single impurity. We approach the model using Wilson Renormalization Group techniques, in which we write the fields appearing in the corresponding Quantum Field Theory as a superposition of fields on different scales, and consider a hierarchical version of the model by assuming that a field on some scale does not fluctuate on smaller scales. Due to the Fermionic nature of the system, the hierarchical model is exactly solvable, in that the renormalization group flow equations are finite. In contrast to non-hierarchical models in which the flow equations are usually written as power series which only converge if the interaction is weak, the finite nature of the flow equations enables us to investigate the non-perturbative regime of the hierarchical model, i.e. when the renormalized interaction is not small. We show that if the interaction is anti-ferromagnetic, then the renormalization group flow tends to a non-Gaussian fixed point, even for very small values of the interaction. This means that the behavior of the interacting system is qualitatively different from the non-interacting (Gaussian) one, which we illustrate by showing that the 0-temperature magnetic susceptibility of the impurity, which is infinite in the non-interacting case, is finite for arbitrarily small anti-ferromagnetic interactions.

Joint work with Giuseppe Benfatto and Giovanni Gallavotti.

Time decay of the wave functions for magnetic Schrödinger operators
Hynek Kovařík

In this talk we will review some recent results concerning dispersive estimates for Schrödinger operators with magnetic field. In particular we will show that, under suitable regularity and decay assumptions on the magnetic and electric field, the behaviour of the corresponding resolvent at the threshold and the long-time decay of the unitary group are determined by the total flux of the magnetic field.

Selfadjoint elliptic operators with boundary conditions on not closed hypersurfaces
Andrea Mantile

The abstract theory of self-adjoint extensions of symmetric operators is used to construct self-adjoint realizations of a second-order elliptic differential operator on $\mathbb{R}^n$ with linear boundary conditions on (a relatively open part of) a compact hypersurface. Our approach allows to obtain Kreĭn-like resolvent formulae where the reference operator coincides with the "free" operator with domain $H^2(\mathbb{R}^n)$; this provides an useful tool for the scattering problem from a hypersurface. Moreover, Schatten-von Neumann estimates, for the difference of the powers of resolvents of the free and the perturbed operators, yield the existence and completeness of the wave operators of the associated scattering systems. As a concrete example, we focus on Robin-type boundary conditions on a $(n-1)$-dimensional sheet with regular boundary.

This is a joint work with A. Posilicano and M. Sini

The analogies between prototypes of periodic Schrödinger operators via Bloch-Floquet methods and the ergodic Laplacian, an ergodic Schrödinger operator
Giovanna Marcelli

After studying separately the Schrödinger operators relative to quantum dynamics in crystalline solids and disorderly crystalline solids, respectively periodic Schrödinger operators and random Schrödinger operators, we will put in evidence the algebraic and hilbertian analogies which emerge from the "diagonalization" via Bloch-Floquet transform of periodic Schrödinger operators and a particular ergodic random Schrödinger operator, the ergodic Laplacian.

The analysis of these operators will be aimed to the determination of their spectrum and spectrum type.

Stability of closed gaps for the alternating Kronig-Penney Hamiltonian
Domenico Monaco

We consider a Kronig-Penney-like model, consisting of periodic equispaced delta interactions with couplings which alternate in sign. The even spectral gaps of this Hamiltonian, corresponding to zero Bloch momentum (at the center of the Brillouin zone), are known to be closed. We prove that this property is stable under approximation via smooth periodic potentials, provided the approximating sequence is chosen suitably. This shows that the closedness of the even gaps is a physically meaningful property, and does not apply just to the idealized model of point-like interactions.

This is joint work with A. Michelangeli (LMU Munich and SISSA, Trieste).

Controllability of spin-boson systems
Lorenzo Pinna

In this talk I will discuss the controllability properties of some spin boson systems, i.e. models of a spin $1/2$ particle in interaction with a particular mode of a quantized bosonic field. I will review some well known results for the so-called Eberly-Law model and for a two-level ion trapped in a harmonic potential. Then I will present a sufficient condition for approximate controllability recently established by U. Boscain et al. in the context of geometric control theory. This spectral condition, based on the non-resonance of the energy levels, can be used to study the controllability of many quantum bilinear systems. I will discuss the application of this criterion to the Jaynes-Cummings model.

NLS ground states on graphs
Enrico Serra

We investigate the existence of ground states for the NLS energy on metric graphs. In particular, we describe sufficient conditions for the existence of ground states and a topological assumption that prevents existence. Next we analyze how the metric properties of the graph, besides its topology, may influence the existence of ground states. In order to obtain the results, we make an extensive use of rearrangement techniques.

NLS equation on metric graphs with localized nonlinearities
Lorenzo Tentarelli

In this talk I will present some recent results on the NLS equation on *metric graphs* with *localized nonlinearities*. In particular, I will focus on two issues arising in the study of *quantum graphs*. The first one concerns existence or nonexistence of *ground states*, namely, minimizers of the NLS energy functional

$$E(u) := \frac{1}{2}\|u'\|^2_{L^2(\mathcal{G})} - \frac{1}{p}\|u\|^p_{L^p(\mathcal{K})} \qquad (2 < p < 6),$$

(where $\mathcal{G}$ is a *non-compact* metric graph with *compact core* $\mathcal{K}$), in the class of functions $u \in H^1(\mathcal{G})$ subject to the *mass constraint*

$$\|u\|^2_{L^2(\mathcal{G})} = \mu.$$

The second one, concerns existence and multiplicity of *bound states* for the NLS equation. These arise as constrained critical points of the functional E at higher levels, that are located by some *min-max* procedure based on the notion of *Krasnosel'skii genus*.

Dirac-Kaehler operator on spheres

Alessandro Zampini

The idea of this talk is to review the construction of the Dirac operator due to Kaehler, and to show some preliminary results that show how this formalism allows to introduce a meaningful Dirac operator on a class of spheres.